The Synthesis of System Designs: I. Elementary Decomposition Theory

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A system synthesis principle is proposed from which processes can be composed to perform an assigned task. Synthesis is performed by the sequential decomposition of the design problem into subproblems which eventually reach the level of available technology.

A prominent feature of the practice of process design is the lack of any theoretical guidance in the synthesis of system structure. The units of available technology, such as dryers, heat exchangers, reactors, distillation towers, and so forth, are assembled in an empirical way to perform processing tasks which are beyond the capacity of any single piece of available technology. A common empirical approach to system synthesis involves the drawing of an analogy between the new processing task and some old processing task for which a process is extant. The new process is then patterned after the old process with no assurance that the optimal structure is obtained.

In this paper we examine a primitive theory of system synthesis which involves the fracturing of a design problem into a sequence of subdesign problems. The efficiency of synthesis by problem decomposition depends on the prior knowledge of points of fracture in a new design problem and on the ability to estimate the economic characteristics of the optimal solution to an unsolved problem. These difficulties limit the primitive theory to certain elementary design problems, and further extensions of the theory, along the lines to be discussed in other papers in this series, are required as more complex design problems are encountered.

In the second paper in this series we shall examine the use of heuristic decision strategies as a basis of system synthesis. These are the same methods which have found use in the development of theorem proving computer programs. The heuristic approach has the advantage that there is no need for prior information on points of problem fracture and has the disadvantage of providing no test of optimality. This then suggests a hybrid method of synthesis based partially on the theory of decomposition to be discussed here and the heuristic decision strategies to be discussed next. The hybrid aproach provides the basis of the third paper on the synthesis of system designs.

DESIGN PROBLEM DECOMPOSITION

Suppose that a processing system is to be composed of, say, one hundred units of available technology, and that attention is severely restricted to designs which exhibit an acyclic structure, no recycle. There are $100! = 10^{150}$ unique process systems which can be thus composed. And, industrial processes commonly achieve great efficiency by the addition of recycle streams, further complicating the problem of synthesis by orders of magnitude. It is imperative that the best arangement of available technology be sought by the most efficient means possible, for otherwise a nonoptimal design will doubtless be obtained.

The experienced process designer may avoid these combinatorial problems empirically by an approach that might be called *process design decomposition*. Rather than focus attention on the details of the design during the initial phases of system synthesis, the design is decomposed into a number of subdesign problems, and the much simpler problem of selecting among the alternative structures at this level is attacked. Then each of the subproblems is decomposed into smaller design problems until the level of available technology is reached. For example, in the synthesis of the design for a refinery, the designer may begin by attacking the gross design problem of synthesizing a system of large subsystems for desulfurizing crude oil, crude oil fractionation, hydrocracking, product blending, and so forth. Once the structure is established involving these subsystems, further decomposition is performed until the level of pumps, heat exchangers, reactors, vessels, extractors, and the like is reached. The theory of problem decomposition is an attempt to formalize this empirical approach to synthesis.

PROBLEM STATEMENT

The system design problem upon which attention is to be focused is now defined.

Task Constraints

The task to be performed by the system shall be defined by constraints on the set of variables **X**:

$$\mathbf{X} = \mathbf{x} \tag{1}$$

For example, the set of variables **X** could be the availability of crude oil, the sulfur content of the crude, the temperature of available cooling water, the desired production of gasoline, and so forth. The set **x** is the specific numerical values of the **X** which obtain in a given problem.

Unsynthesized System

An unsynthesized system shall be denoted by an empty box into which the task constraints **X** enter as arrows, see Figure 1.

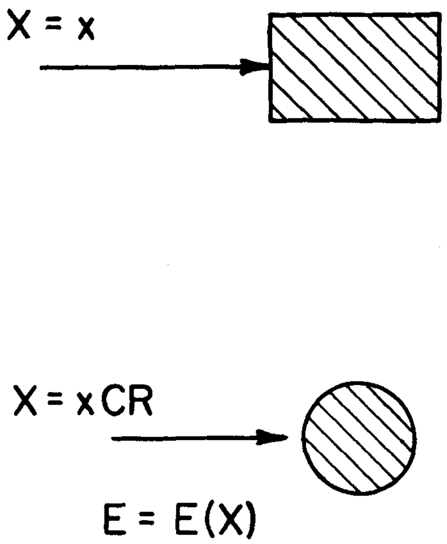

Fig. 1.

Available Technology

If the task constraints which define a given design problem fall within a well-defined region of available technology R, the task is identified as one that can be handled by available equipment, and no synthesis problem exists:

$$\mathbf{X} \subset R \qquad (2)$$

Should (2) be satisfied, the box denoting the unsynthesized system is replaced by a circle which denotes available technology.

The Economics of Available Technology

Should a task be within the region of available technology [that is, (2) is satisfied], a cost or profit of accomplishing the task is assumed known. This is the economics E of available technology:

$$E = E(\mathbf{X}) \quad \text{for } \mathbf{X} \subset R \qquad (3)$$

For example, if the constraints $\mathbf{X}$ define a heat transfer task which is within the region of existing heat transfer technology, the economics E might be the cost of the heat exchanger required to perform the task.

The Synthesis Objective

Should the task constraints $\mathbf{X}$ not satisfy (2), the task cannot be performed by one piece of existing technology, and it is necessary to synthesize a system. The objective of synthesis is to select and arrange the technology so as to optimize the total economics of the system:

$$O^*(\mathbf{X}) = \text{Opt} \left\{ \sum E_j (\mathbf{X}_j) \right\} \qquad (4)$$

Each of the subtasks j in (4) must be within existing technology

$$\mathbf{X}_j \subset R \quad \text{for all } \mathbf{X}_j \qquad (5)$$

A PRIMITIVE THEORY

We shall now describe a method of synthesizing the solution to the design problem stated by (1) through (5). The original design problem defined by the task constraints $\mathbf{X}$ shall be decomposed into two smaller problems $\mathbf{S}_I$ and $\mathbf{S}_{II}$, where

$$\begin{aligned} \mathbf{S}_I &= \mathbf{X}_I \cup \mathbf{T} \\ \mathbf{S}_{II} &= \mathbf{X}_{II} \cup \mathbf{T} \\ \mathbf{X} &= \mathbf{X}_I \cup \mathbf{X}_{II} \\ \mathbf{X}_I \cap \mathbf{X}_{II} &= O \end{aligned} \qquad (6)$$

$\mathbf{T}$ is a set of artificially imposed tear constraints which unite the two subtasks $\mathbf{S}_I$ and $\mathbf{S}_{II}$ to accomplish the original task defined by $\mathbf{X}$.

For example, in Figure 2 we have an initial task defined by two constraints $\mathbf{X} = (X_1, X_2)$, and the unsynthesized system is decomposed into two unsynthesized subsystems I and II by the tear constraints $\mathbf{T} = (t_1\, t_2)$. Notice that there are several ways in which the constraints $\mathbf{T}$ can be used to decompose this initial problem. The selection between these alternative structures constitutes the beginning step in system synthesis.

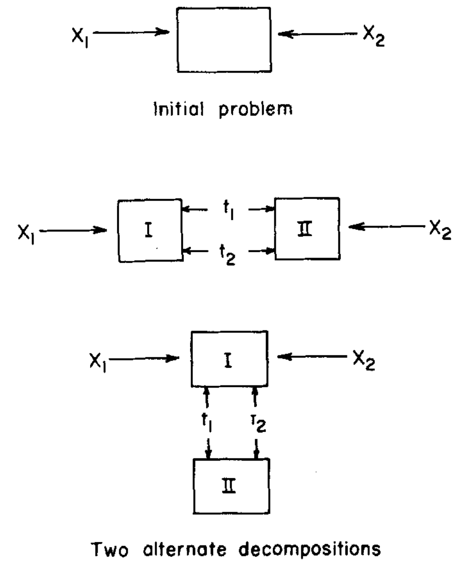

Fig. 2. Problem decomposition.

The basic problem we now discuss is that of selecting between the alternate structures which arise in the decomposition of a task into two subtasks, for if this can be solved, the entire system can be synthesized merely by sequentially decomposing the subtasks to such an extent that existing technology is reached. That is, when (7) is satisfied

$$\mathbf{S}_j \subset R \qquad (7)$$

A subtask $\mathbf{S}_j$ is identified as existing technology.

The inspection of Equation (4) indicates how the selection between alternative decomposition structures can be accomplished, for this equation can be written as

$$O^*(\mathbf{X}) = \underset{\substack{\text{tear location} \\ \text{tear value}}}{\text{Opt}} \{O(\mathbf{S}_I) + O(\mathbf{S}_{II})\}$$

$$= \underset{\substack{S_I \text{ and } S_{II} \\ \text{Selection}}}{\text{Opt}} \{\underset{\mathbf{T}}{\text{Opt}} [O^*(S_I) + O^*(S_{II})]\} \qquad (8)$$

The original task $\mathbf{X}$ is divided into tasks $\mathbf{X}_I \cup T$ and $\mathbf{X}_{II} \cup T$, and the terms $O^*(X_I \cup T)$ and $O^*(X_{II} \cup T)$ are the optimal objective functions that can be obtained by the solution to the subtasks, given the values of $\mathbf{X}_I \cup \mathbf{T}$ and $\mathbf{X}_{II} \cup \mathbf{T}$. The tear constraints $\mathbf{T}$ are free to be adjusted, and the interior optimization merely adjusts $\mathbf{T}$ to optimize the sum of the optimum objective function of the two parts I and II. The exterior optimization is over the distribution of $\mathbf{X}$ between $\mathbf{X}_I$ and $\mathbf{X}_{II}$ and constitutes an optimization over the structure of the system.

In Figure 2, (8) reduce to (9):

$$O^*(x_1, x_2) = \text{Opt} \begin{cases} \underset{t_1 t_2}{\text{Opt}} [O^*(x_1 t_1 t_2) + O^*(x_2 t_1 t_2)] \\ \underset{t_1 t_2}{\text{Opt}} [O^*(x_1 x_2 t_1 t_2) + O^*(t_1 t_2)] \end{cases} \qquad (9)$$

The implementation of (8) requires a knowledge of the optimal objective function that can be obtained for any task, and this information is only available for tasks within the region of existing technology; then (10) holds:

$$O^*(S_j) = E(S_j) \qquad (10)$$

when $S_j C R$.

To the extent that the optimal objective function is known, synthesis would proceed by Figure 3.

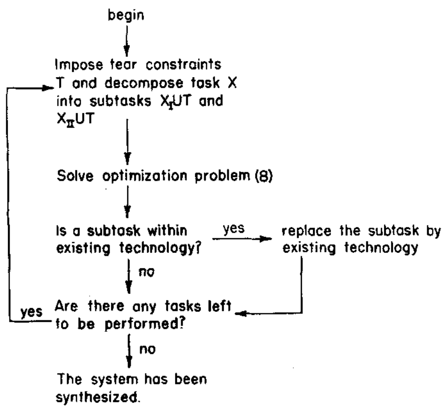

Fig. 3. The synthesis algorithm.

THE ACCUMULATION OF DESIGN EXPERIENCE

It is clear that optimal objective function $O^*(\mathbf{X})$ cannot be known for a new process design problem, since O^* is the economic objective function for the optimal solution to a problem the solution of which is being sought. The synthesis algorithm, Figure 3, cannot be implemented. However, in a given area of processing technology there will be sufficient experience available in the form of previously solved designs to construct an estimated optimal objective function $O^{(1)}(X)$.

Such an estimated optimal objective function can be used in the synthesis algorithm to compose an estimated optimal system, which then forms the basis for the improvement of initial estimate $O^{(1)}$ to form $O^{(2)}$, an improved estimate of the optimal objective function O^*. The iteration on the optimal objective function with the synthesis algorithm used may converge to the synthesis of the optimal system as the sequence $O^{(1)}$, $O^{(2)}$, $O^{(3)}$ . . . converges to O^*. The iteration plan is outlined in Figure 4.

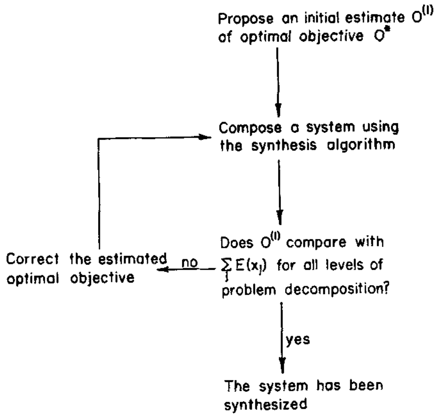

Fig. 4. The accumulation of experience.

We shall now examine the two kinds of synthesis errors that will arise from the use of an estimated optimal objective function, and we shall discuss qualitatively means for the elimination of these errors. The convergence of the problem decomposition theory of system synthesis depends critically on the means by which these errors can be eliminated by the proper accumulation of design experience.

The use of an estimated optimal objective function in the synthesis algorithm will lead to the synthesis of a system which deviates from optimality in one or both of two ways:

1. The tear constraint sets $\mathbf{T}$ have not been adjusted to their optimal numerical values.
2. The tear constraint sets $\mathbf{X}_I \cup \mathbf{T}$ and $\mathbf{X}_{II} \cup \mathbf{T}$ do not decompose the assigned problem into the optimal arrangements of existing technology.

Both of these errors in synthesis will express themselves by an observed difference between the estimated optimal objective function, $O^{(1)}$ and the actual objective function of the composed system O_{act}.

$$O_{\text{act}}(X) = \sum_j E(X_j) \qquad (11)$$

where $X_j C R$

The first kind of error is of minor consequence, since it does not involve an erroneous system structure; it only involves an erroneous system optimization Thus, the application of the standard methods of process optimization as outlined in reference *1* will remove the first class of errors, yielding a partially optimized actual function O^*_{act}:

$$O^*_{\text{act}} = \underset{\mathbf{T}}{\text{Opt}} \left\{ \sum E(X_j) \right\} \qquad (12)$$

Equation (12) achieves optimization only to the extent that the nonstructural design errors are removed. A failure of O_{act} and O^*_{act} to coincide with $O^{(1)}$ at all phases of decomposition indicates a need for further iteration.

The improved estimated optimal objective function $O^{(2)}(X)$ may be obtained by an analysis of the design errors which express themselves by the failure of $O^{(1)}(X)$ to map into $O_{\text{act}}(X)$ and $O^*_{\text{act}}(X)$.

Rather than continue further with theoretical conjecture at this point, we shall apply the elementary theory of synthesis by problem decomposition to a simplified process design problem. The points outlined above will be explained by example.

THE SYNTHESIS OF A HEAT EXCHANGE SYSTEM

In this section we shall examine the synthesis of a system of heat exchangers which economically transfer heat from a hot fluid to a cold fluid. This kind of a problem arises frequently in process engineering, say in the preheating of the feed to distillation systems. However, we shall take the liberty of simplifying greatly the model of a heat exchanger to focus attention on the principles of synthesis and eliminate unnecessary detail.

The Design Problem

A hot fluid stream available at 500°F. and at a rate of F lb./hr. must be cooled to 300°F. by the exchange of heat with a cool fluid available at 0°F. and at a rate of F lb./hr.

Existing Technology

The existing technology takes the form of heat exchangers which are only capable of increasing the temperature of the one fluid by 50°F. and decreasing the temperature of the other fluid by a similar amount. This constraint on existing technology may be thought to arise from heat exchange surface fouling or some other technical phenomenon:

$$\Delta T_H = \Delta T_C = 50°\text{F.} \tag{14}$$

Thus, the duty Q B.t.u./hr. of a given exchanger is fixed at

$$Q = FCp\,50 \tag{15}$$

The area A sq. ft. of the exchanger required to accomplish this transfer of heat is given by

$$Q = UA \max\{T_H - T_C\} \tag{16}$$

Furthermore, a minimum approach temperature of 10°F. is imposed:

$$\min\{T_H - T_C\} = 10°\text{F.} \tag{17}$$

Economics of Existing Technology

The cost C of a heat exchanger is correlated to the square root of the exchanger surface area of the exchanger:

$$C = K(A)^{1/2} \quad \$/\text{year} \tag{18}$$

This in turn is related to the max $[T_H - T_C]$; thus

$$C = K\left(\frac{Q}{U \max[T_H - T_C]}\right)^{1/2}$$

$$C = \frac{K'}{(\max[T_H - T_C])^{1/2}} \tag{19}$$

where K' is a known constant for the existing technology.

The Design Objective

The objective of the system design problem is to select a number of the exchangers described above and connect them into a system so as to minimize the total cost of the heat exchange task:

$$O^* = \min\left\{\sum_j C_j\right\} \tag{20}$$

$$O^* = K' \min \sum_j \frac{1}{(\max[T_{Hj} - T_{Cj}])^{1/2}} \tag{21}$$

This then describes the design task, as summarized in Figure 5.

To begin the synthesis of a system, it is necessary to estimate O^* for any combination of available hot fluid temperatures T_H and cold fluid temperatures T_C. Our initial estimate $O^{(1)}$ shall be a simple extension of (19):

$$O^{(1)} = \min\left[\frac{K'}{(\max[T_H - T_C])^{1/2}}\right] \tag{22}$$

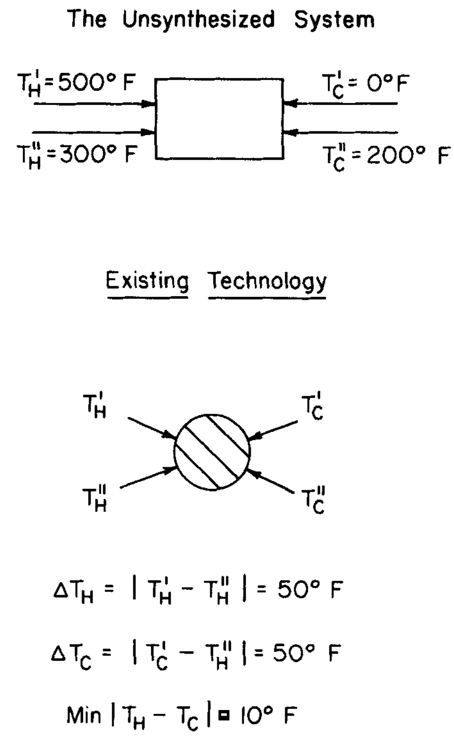

Fig. 5.

The decomposition shall be effected by two tear constraints T_H and T_C, as illustrated in Figure 6, subject to the constraint that

$$\min[T_H - T_C] = 10°\text{F.} \tag{23}$$

for any subtask resulting from decomposition.

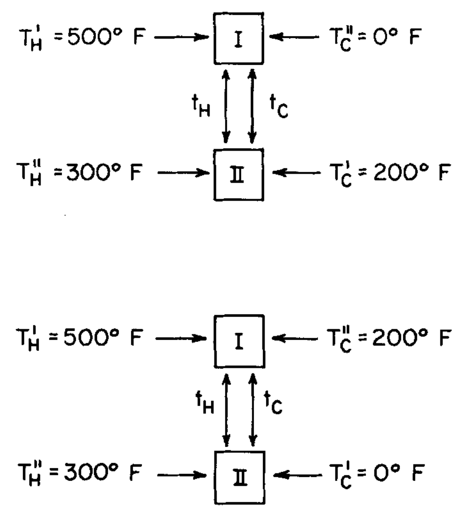

Fig. 6. Problem decomposition by two tear constraints.

The selection between the decomposition alternatives is made through the optimization problem

$$\min \begin{cases} \min\limits_{t_H,\, t_C} \{O^{(1)}(500, t_H, 0, t_C) + O^{(1)}(t_H, 300, t_C, 200)\} \\ \min\limits_{t_H,\, t_C} \{O^{(1)}(500, t_H, 200, t_C) + O^{(1)}(t_H, 300, t_C, 0)\} \end{cases} \tag{24}$$

where in $O^{(1)}$ $(T_H', T_H'', T_C', T_C'')$ the first two entries refer to the two temperature constraints on the hot fluid, and the last two entries refer to the two cold fluid constraints.

The first minimization problem in (24) is solved below:

$$\min_{t_H,\, t_C} \frac{K'}{(\max\{|500-0|, |500-t_C|, |t_H-0|, |t_H-t_C|\})^{1/2}} + \frac{K'}{(\max\{|300-t_C|, |t_H-t_C|, |300-200|, |t_H-200|\})^{1/2}} \quad (25)$$

This reduces to (26) since $500 > t_H > 300$ and $200 > t_C > 0$:

$$\min_{t_H,\, t_C} \left\{ \frac{K'}{(500)^{1/2}} + \frac{K'}{(t_H - t_C)^{1/2}} \right\} \quad (26)$$

The minimum cost is reached when $t_H - t_C$ is at its maximum, namely, when $t_H = 450°$F. and $t_C = 50°$F., and when task I is identified as existing technology. Thus, the minimum of (26) is

$$K' \left[\frac{1}{(500)^{1/2}} + \frac{1}{(400)^{1/2}} \right] = K'\, 0.095 \quad (27)$$

The second minimization in (24) is shown below:

$$\min_{t_H,\, t_C} \left\{ \frac{K'}{(\max\{|500-200|, |500-t_C|, |t_H-200|, |t_H-t_C|\})^{1/2}} + \frac{K'}{(\max\{|300-t_C|, |300-0|, |t_H-t_C|, |t_H-0|\})^{1/2}} \right\} \quad (28)$$

This reduces to (29) from $500 > t_H > 300$ and $200 > t_C > 0$:

$$\min_{t_H,\, t_C} \left\{ \frac{K'}{(500 - t_C)^{1/2}} + \frac{K'}{(t_H - 0)^{1/2}} \right\} \quad (29)$$

The minimum of (29) is reached when $t_H = 400°$F. and $t_C = 100°$F., yielding a minimum cost shown in (30):

$$K' \left[\frac{1}{(400)^{1/2}} + \frac{1}{(400)^{1/2}} \right] = K'\, 0.10 \quad (30)$$

A comparison of (27) and (30) solves the structural minimization problem in (24), and the decomposition shown in Figure 7 is thus recommended.

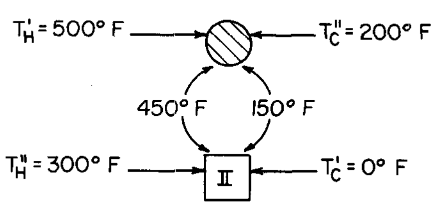

Fig. 7. The results of the first decomposition with $O^{(1)}$ used.

There remains a task beyond existing technology, task II, that of cooling the hot fluid from 450° to 300°F. by using a cool fluid at 50°F. which shall be heated to 200°F. By analogy to this first decomposition problem, the synthesis algorithm will compose the cocurrent heat exchanger structure shown in Figure 8. The task constraints have been replaced by material flow.

In Table 1 a comparison is made between the estimated optimal objective function (22) which was used to accomplish the synthesis and the actual objective function which occurs in the synthesized system. A wide divergence occurs, indicating that false information was used to achieve synthesis and that the system shown in Figure 8 is nonoptimal. Notice that $O^{(1)}$ predicts in Table 1 that the smaller the task, the more costly the system.

Notice in Table 1 that the estimated optimal objective function (22) consistently underestimates the cost of accomplishing a heat exchange task, and that the farther the task is from existing technology, the worse the estimate. This suggests that (22) be altered to include some expression of the extent to which a task deviates from existing technology. This might lead to the new estimated optional objective function shown in (31):

$$O^{(2)} = \left| \frac{T_H' - T_H''}{50} \right|^{1.2} \left\{ \min \frac{K'}{(\max[T_H - T_C])^{1/2}} \right\} \quad (31)$$

Table 1. Comparison of Estimated Objective Function and Actual Objective Function for Figure 8

Decomposition groups in Figure 8	$O^{(1)} = \dfrac{K'}{(\max\,[T_H - T_C])^{1/2}}$	$\sum C_j = \sum_j \dfrac{K'}{(\max\,(T_{Hj} - T_{Cj}))^{1/2}}$
1, 2, 3, 4	$\dfrac{K'}{(500)^{1/2}} = K'\, 0.046$	$K'\left[\dfrac{1}{(500)^{1/2}} + \dfrac{1}{(400)^{1/2}} + \dfrac{1}{(300)^{1/2}} + \dfrac{1}{(200)^{1/2}}\right] = K'\, 0.225$
2, 3, 4	$\dfrac{K'}{(400)^{1/2}} = K'\, 0.050$	$K'\left[\dfrac{1}{(400)^{1/2}} + \dfrac{1}{(300)^{1/2}} + \dfrac{1}{(200)^{1/2}}\right] = K'\, 0.179$
3, 4	$\dfrac{K'}{(300)^{1/2}} = K'\, 0.058$	$K'\left[\dfrac{1}{(300)^{1/2}} + \dfrac{1}{(200)^{1/2}}\right] = K'\, 0.129$
4	$\dfrac{K'}{(200)^{1/2}} = K'\, 0.071$	$\dfrac{K'}{(100)^{1/2}} = K'\, 0.071$

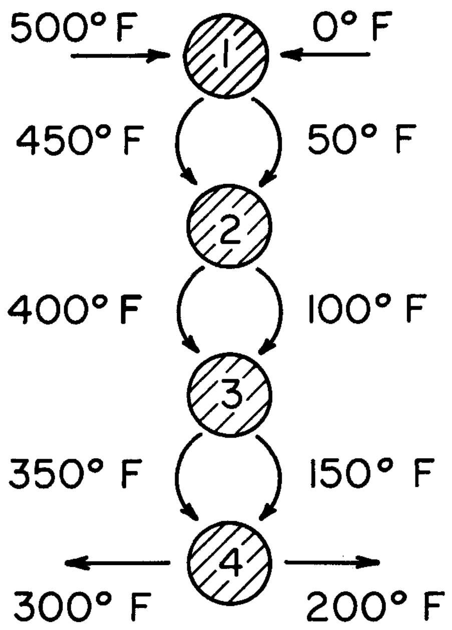

Fig. 8. The system synthesized with $O^{(1)}$ used.

In (31) $O^{(1)}$ is multiplied by the factor $\left| \dfrac{T_H' - T_H''}{50} \right|^{1.2}$ which is the number of 50°F. temperature units needed to span the task of cooling the fluid by $|T_H' - T_H''|$ °F. raised to a power greater than 1 to account for the fact that all units will not have the advantage of the maximum temperature difference. This is a more realistic estimate of the optimal achievable objective function O^*.

The transition from $O^{(1)}$ to $O^{(2)}$ was achieved by empirical methods, and there is a need for further research on methods which best incorporate experience gained during interations through the synthesis algorithm.

Synthesis then proceeds in this manner. The first optimization problem in (24) is shown in (32):

$$\min_{t_H,\, t_C} \left\{ K' \left| \frac{500 - t_H}{50} \right|^{1.2} \left(\frac{1}{(500)^{1/2}} \right) + K' \left| \frac{t_H - 300}{50} \right|^{1.2} \left(\frac{1}{(t_H - t_C)^{1/2}} \right) \right\} \qquad (32)$$

The minimum of (32) occurs at $t_H = 450$ and $t_C = 50$, yielding the minimum cost of

$$K' \left(\frac{1}{(500)^{1/2}} + \frac{3^{1.2}}{(400)^{1/2}} \right) = K' \ (0.232) \qquad (33)$$

The second minimization problem in (24) is shown in (34):

$$\min_{t_H,\, t_C} \left\{ K' \left| \frac{500 - t_H}{50} \right|^{1.2} \left(\frac{1}{(500 - t_C)^{1/2}} \right) + K' \left| \frac{t_H - 300}{50} \right|^{1.2} \left(\frac{1}{(t_H - \mathrm{O})^{1/2}} \right) \right\} \qquad (34)$$

The minimum of (34) occurs at $t_H = 450$ and $t_C = 150$, yielding a minimum cost of

$$K' \left(\frac{1}{(350)^{1/2}} + \frac{3^{1.2}}{(450)^{1/2}} \right) = K' \ (0.230) \qquad (35)$$

Now a comparison of (33) and (35) shows that the second problem dominates, and the partial structure in the Figure 9 results.

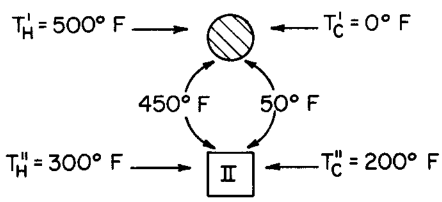

Fig. 9. Results of the first decomposition with $O^{(2)}$ used.

There remains a task beyond existing technology, task II, that of cooling the hot fluid from 450° to 300°F. by using a cool fluid at 0°F. which shall be heated to 150°F. By analogy to this first decomposition problem, the synthesis algorithm might compose the countercurrent heat exchanges structure shown in Figure 10.

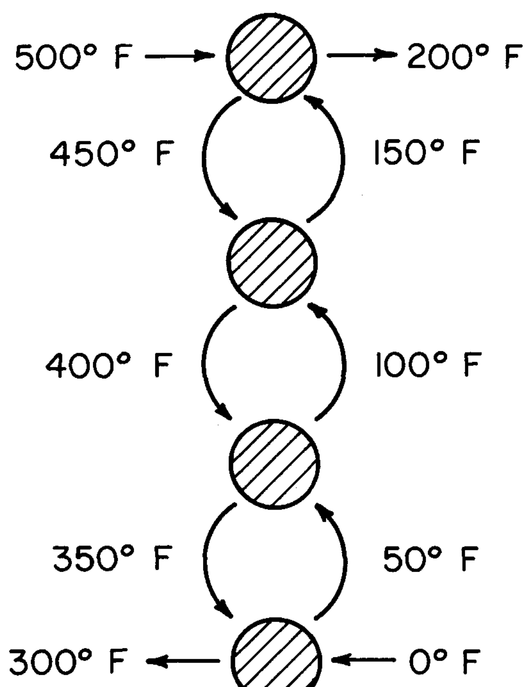

Fig. 10. The system synthesized with $O^{(2)}$ used.

Table 2. Comparison of the Estimated Optimal Objective Function and the Actual Objective Function for Figure 10

Decomposition groups in Figure 10	$O^{(2)} = \left\| \frac{T_H' - T_H''}{50} \right\|^{1.2} \frac{K'}{(\max\{T_H - T_C\})} \frac{1}{2}$	$\sum C_j = \sum_j \frac{K}{(\max (T_{Hj} - T_{Cj}))^{1/2}}$
1, 2, 3, 4	$K' \left(\frac{200}{50}\right)^{1.2} \left(\frac{1}{(500)^{1/2}}\right) = K'\ (0.238)$	$K' \left(\frac{1}{(350)^{1/2}} + \frac{1}{(350)^{1/2}} + \frac{1}{(350)^{1/2}} + \frac{1}{(350)^{1/2}}\right) = K'\ (0.214)$
2, 3, 4	$K' \left(\frac{150}{50}\right)^{1.2} \left(\frac{1}{(450)^{1/2}}\right) = K'\ (0.226)$	$K' \left(\frac{1}{(350)^{1/2}} + \frac{1}{(350)^{1/2}} + \frac{1}{(350)^{1/2}}\right) = K'\ (0.161)$
3, 4	$K' \left(\frac{100}{50}\right)^{1.2} \left(\frac{1}{(400)^{1/2}}\right) = K'\ (0.201)$	$K' \left(\frac{1}{(350)^{1/2}} + \frac{1}{(350)^{1/2}}\right) = K'\ (0.106)$
4	$K' \left(\frac{50}{50}\right)^{1.2} \left(\frac{1}{(350)^{1/2}}\right) = K'\ (0.050)$	$K' \left(\frac{1}{(350)^{1/2}}\right) = K'\ (0.059)$

In Table 2, the estimated optimal objective function is compared with the actual objective function. There exists a closer match with $O^{(2)}$ used than occurred in Figure 9. The system in Figure 10 is more nearly optimal than the system in Figure 8; the minimum costs are K' 0.214 and K' 0.225, respectively.

CONCLUDING REMARKS

We have presented an elementary theory of system synthesis and illustrated how different system structures are composed as synthesis proceeds towards the optimal system.

However, there is much more that needs to be reported before the industrial significance of problem decomposition can be seen. The following questions need be answered.

How best can one select the tear constraint set **T** in a new and unfamiliar design problem?

Is it possible to estimate the optimal objective function for new and unfamiliar design problems?

How best can the estimated optimal objective functions be improved?

Can industrial design problems be relegated in part to the computer with these synthesis algorithms?

The problem of efficiently detecting tear location is closely related to the problem of system decomposition which has received considerable attention in the recent literature (*2*). The estimation of the optimal objective function for a new and unfamiliar problem is precisely the problem the cost estimator faces in the initial phases of design (*3*). The problem of improving the estimate of the optimal objective function from data gathered by an initial synthesis is a new area of research. The primitive theory of problem decomposition stands to gain in efficiency as these questions are examined in detail.

ACKNOWLEDGMENT

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NOTATION

$\mathbf{A} \subset \mathbf{B}$ = set **A** is contained in set **B**
$\mathbf{A} \cup \mathbf{B}$ = union of sets **A** and **B**
$\mathbf{A} \cap \mathbf{B}$ = intersection of sets **A** and **B**
C_j = cost of j[th] exchanger, given by (19)
E = economic measure of performance
$E(\mathbf{X}_j)$ = economics of existing technology by task $\mathbf{X}_j$
$O^{(i)}(\mathbf{X})$ = estimate of the objective function achievable for task **X**
$O^*(\mathbf{X})$ = optimal objective function for task **X**
S = subtask set
T = tear constraint set
T_H, T_C = temperatures, °F.
t_H, t_C = tear constraint temperatures, °F.
U = heat transfer coefficient, B.t.u./(hr.) (sq. ft.)
X = task constraint set
$\mathbf{X}_j$ = subtask of technology j
$\mathbf{X}_{\mathrm{I}}, \mathbf{X}_{\mathrm{II}}$ = disjoint subsets of task constraints

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